Using Cayley graphs in costruction of large scale computer networks

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Nowadays, large-scale computing systems – such as supercomputers, cloud platforms, data centers, and parallel processing systems – can contain thousands or even millions of interconnected processors. Designing such networks requires careful attention to latency, efficiency, scalability, and fault tolerance, thus the communication network architecture is critical. One of the central challenges in this domain is minimizing communication delays while limiting the number of direct connections per node, since each connection incurs both physical and management costs.

The degree-diameter problem is defined as follows: given diameter d and maximum degree k, determine a graph with largest possible number of vertices satisfying these constraints. Graph theory provides a rigorous framework for modeling networks, where vertices correspond to computing devices and edges represent communication links. The degree-diameter problem is therefore particularly relevant in this context, as it captures the trade-off between node degree and communication efficiency.

For given finite group Γ and a unit-free inverse-closed generating set X, the vertex set of the Cayley graph $Cay(\Gamma, X)$ is Γ and there is an edge between g and gx for each $g \in \Gamma$ and $x \in X$. This paper examines the application of Cayley graphs to the construction of efficient large-scale computer networks. Cayley graphs, derived from group-theoretic principles, are vertex-transitive and regular, and they lend themselves to algebraic analysis. These properties make them attractive for designing network topologies with predictable, uniform behavior and efficient routing. We survey known results on small-diameter Cayley graphs and illustrate how they can be used to model interconnection networks that offer advantages such as symmetry, fault tolerance, and simplicity of routing algorithms.

We present several examples of modeling computing systems using Cayley graphs, focusing on key structural properties that make them suitable for large-scale networks. In particular, we highlight the use of diameter-two Cayley graphs (for example as described in [1] in the construction of networks that are both scalable and efficient.

[1] M. Abas, *Cayley graphs of diameter two with order greater than 0.684 of the Moore bound for any degree*, European Journal of Combinatorics, **57**, (2016), 109–120